

Adaptive Beamforming Algorithms for Anti-Jamming

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Abstract

Smart antennas possess the capability of suppressing jamming signal, so they can improve signal to interference plus noise ratio (SINR). Array processing utilizes information regarding locations of signal to aid in interference suppression and signal enhancement and is considered promising technology for anti-jamming. In this paper we studied three beamforming algorithms, Least Mean Square (LMS) algorithm, Optimized-LMS algorithm and Recursive Least Squares (RLS) algorithm. Simulation results are presented to compare the ability of these three algorithms to form beam in the direction of desired signal and place null in the direction of interference signal. Dependency of these algorithm on SNR and SIR is also analyzed. It has been found that RLS algorithm is best suited for anti-jamming applications.

Keywords: Adaptive Array Signal Processing, Anti-Jamming, Optimized LMS Algorithm

1. Introduction

Potential jamming in military and critical civilian applications has been a major concern for system designers. And usual filtering techniques are not helpful as the jamming signal and desired signal are of same frequency. Various methods have been adopted to avoid jamming, including frequency hopping but it requires excessive bandwidth. Spatial filtering can solve the problem [15] over head without the need of additional bandwidth as signals are filtered on basis of their direction of arrival.

Non-blind algorithms as discussed in this paper require the information of desired signal but blind algorithms such as Constant Modulus Algorithm (CMA) and MUSIC algorithm can estimate the Direction of Arrival (DOA) of the source signal, and then this direction information can be utilized in non-blind beamforming algorithms to form beam in the estimated direction.

LMS algorithm is known for its simplicity and robustness. The computation complexity of LMS algorithm is $O(M)$ [6]. While it lacks in convergence speed several modifications to the algorithm are proposed including Optimized-LMS [2] Variable Step Size LMS (VSS-LMS) algorithms [12, 13, 1, 3], variable-length LMS algorithm [12], transform domain algorithms [11], and recently CSLMS algorithm [9].

Optimized-LMS algorithm modifies the conventional LMS algorithm with optimized step size and is studied in detail in this paper. The computational complexity of Optimized-LMS is also $O(M)$.

RLS algorithm usually converges with order of magnitude faster than LMS algorithm but the price paid is added complexity. Several variants of RLS algorithm are also proposed on of which is GVFF(Gradient based Variable Forgetting Factor)-RLS) [10]. The complexity of RLS algorithm is $O(M^2)$.

2. Adaptive Beamforming Algorithms

2.1. Least Mean Square (LMS) Algorithm

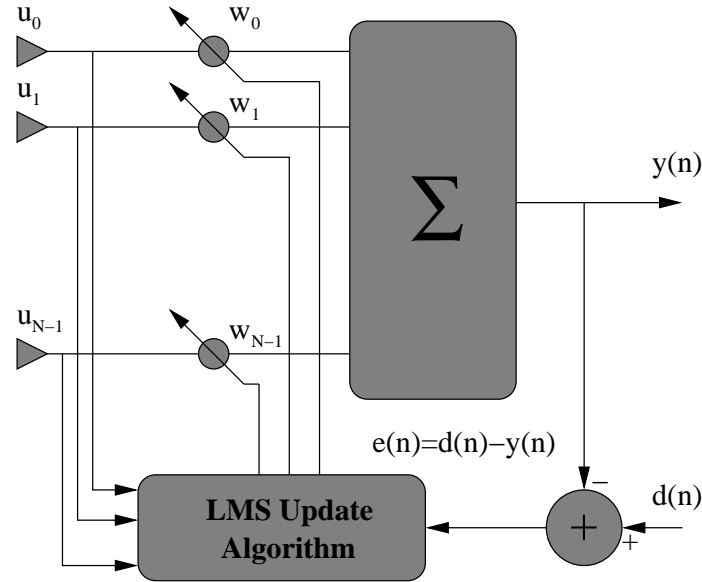


Figure 1. Least Mean Square (LMS) Algorithm

In Fig. 1 the outputs of the individual sensors are linearly combined after being scaled with corresponding weights optimizing the antenna array to have maximum gain in the direction of desired signal and nulls in the direction of interferers.

For beamformer the output at any time n , $y(n)$ is given by a linear combination of the data at M antennas, with $\mathbf{u}(n)$ being the input vector and $\mathbf{w}(n)$ being the weight vector.

$$y(n) = \mathbf{w}^H(n) \mathbf{u}(n) \quad (1)$$

where

$$\mathbf{w}(n) = \sum_{n=0}^{M-1} w_n \quad (2)$$

and

$$\mathbf{u}(n) = \sum_{n=0}^{M-1} u_n \quad (3)$$

The LMS algorithm avoids matrix inverse operation and uses the instantaneous gradient vector $\nabla J(n)$ for weight vector upgradation. The weight vector at time $n + 1$ can be written as

$$\mathbf{w}(n + 1) = \mathbf{w}(n) + \frac{1}{2} \mu [-\nabla J(n)] \quad (4)$$

where μ is the step size parameter which controls the speed of convergence and it lies between 0 and 1. Very small values of μ result in slow convergence and good approximation of the cost function; on the contrary large values of μ may lead to a faster convergence but the stability around a minimum value may be lost.

$$0 < \mu < \frac{1}{\lambda_{max}} \quad (5)$$

An exact calculation of instantaneous gradient vector $\nabla J(n)$ is not possible as prior information of covariance matrix \mathbf{R} and cross-correlation vector \mathbf{p} is needed. So an instantaneous estimate of gradient vector $\nabla J(n)$

$$\nabla J(n) = -2\mathbf{p}(n) + 2\mathbf{R}(n)\mathbf{w}(n) \quad (6)$$

where

$$\mathbf{R}(n) = \mathbf{u}(n)\mathbf{u}^H(n) \quad (7)$$

and

$$\mathbf{p}(n) = d^*(n)\mathbf{u}(n) \quad (8)$$

By putting values from (6), (7), (8) in (4) the weight vector is found to be

$$\begin{aligned} \mathbf{w}(n+1) &= \mathbf{w}(n) + \mu[\mathbf{p}(n) - \mathbf{R}(n)\mathbf{w}(n)] \\ &= \mathbf{w}(n) + \mu\mathbf{u}(n)[d^*(n) - \mathbf{u}^H(n)\mathbf{w}(n)] \\ &= \mathbf{w}(n) + \mu\mathbf{u}(n)e^*(n) \end{aligned} \quad (9)$$

So the LMS algorithm can be described by the three equations as given below

$$y(n) = \mathbf{w}^H(n)\mathbf{u}(n) \quad (10)$$

$$e(n) = d(n) - y(n) \quad (11)$$

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu\mathbf{u}(n)e^*(n) \quad (12)$$

The response of LMS algorithm is determined by three principal factors step size parameter , number of weights and eigenvalue of the correlation matrix of input data vector.

2.2. Optimal LMS Algorithm with Optimal Step Size

The Optimal Robust Adaptive LMS Algorithm without Adaptation Step-Size utilizes input data and error to find the optimum step-size where as Conventional LMS algorithm uses a predetermined step-size. The computational complexity of the proposed algorithm remains the same as of the conventional LMS. As weight update equation of conventional LMS algorithm is

$$w(n+1) = w(n) + 2\mu u(n)e^*(n) \quad (13)$$

$\mathbf{w}(n)$ is the weight vector, $\mathbf{u}(n)$ is the input data vector, $e(n)$ is the error. Cost function can be represented as

$$\begin{aligned} J_{LMS} &= \arg|e|^2_{min} \\ E[|e(n)|^2] &= E[|d(n) - y(n)|^2] \\ &= E\{[d(n) - y(n)][d(n) - y(n)]^*\} \end{aligned} \quad (14)$$

For simplicity, eliminating time index and expected value notation

$$\begin{aligned} |e|^2 &= |d - y|^2 \\ |e|^2 &= (d - y)(d - y)^* \end{aligned} \quad (15)$$

Putting $y = \mathbf{w}^H \mathbf{u}$ in (15)

$$J_{LMS} = (d - \mathbf{w}^H \mathbf{u})(d - \mathbf{w}^H \mathbf{u})' \quad (16)$$

using (13) in (16) gives

$$J_{LMS} = |d|^2 - d\mathbf{u}^H(\mathbf{w} + 2\mu\mathbf{u}e^*) - (\mathbf{w} + 2\mu\mathbf{u}e^*)\mathbf{u}d^* \\ + (\mathbf{w} + 2\mu\mathbf{u}e^*)^H\mathbf{u}\mathbf{u}^H(\mathbf{w} + 2\mu\mathbf{u}e^*) \quad (17)$$

To determine the optimum step-size, differentiate the equation and equate the result to zero.

$$\begin{aligned} \frac{\partial J_{LMS}}{\partial \mu} &= -2\mathbf{u}^H\mathbf{u}de^* + 8\mu\mathbf{u}^H\mathbf{u}\mathbf{u}^H\mathbf{u}ee^* - 2\mathbf{u}^H\mathbf{u}de^* \\ &\quad + 2\mathbf{w}^H\mathbf{u}\mathbf{u}^H\mathbf{u}e^* + 2\mathbf{u}^H\mathbf{u}\mathbf{u}^H\mathbf{w}e^* \\ \frac{\partial J_{LMS}}{\partial \mu} &= -4\Re(\mathbf{u}^H\mathbf{u}de^*) + 4\Re(\mathbf{u}^H\mathbf{u}ye^*) \\ &\quad + 8\mu\mathbf{u}^H\mathbf{u}\mathbf{u}^H\mathbf{u}ee^* \\ \frac{\partial J_{LMS}}{\partial \mu} &= 0 \end{aligned} \quad (18)$$

Where $\Re(X)$ is the real part of X . Equating (18) equal to zero gives

$$\mu_{opt} = \frac{1}{2\mathbf{u}^H\mathbf{u}} \quad (19)$$

Putting value of μ_{opt} in (13) gives

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \frac{\mathbf{u}(n)}{\mathbf{u}(n)\mathbf{u}^H(n)}e^*(n) \quad (20)$$

(20) can be simplified to

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \frac{e^*(n)}{\mathbf{u}^H(n)} \quad (21)$$

A small positive value ϵ is added to the denominator in (21) to make the algorithm robust, especially when the value of the instantaneous sample of the input vector goes to zero.

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \frac{e^*(n)}{\mathbf{u}^H(n) + \epsilon} \quad (22)$$

Empirical results have shown that a step size parameter is still needed for the algorithm to converge. So the weight upgradation equation can be written as

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu \frac{e^*(n)}{\mathbf{u}^H(n) + \epsilon} \quad (23)$$

2.3. Recursive Least Square Algorithm

The autocorrelation of the tap input vector $\mathbf{u}(n)$ of order M-by-M is given by

$$\Phi(n) = \sum_{i=1}^n \lambda^{n-i} \mathbf{u}(i)\mathbf{u}^H(i) + \delta\lambda^n \mathbf{I} \quad (24)$$

λ is a positive constant and is called forgetting factor, δ is the Regularizing term. M-by-1 cross-correlation matrix of the tap input vector $\mathbf{u}(n)$ and desire response $d(n)$ is given by

$$\mathbf{z}(n) = \sum_{i=1}^n \lambda^{-1} \mathbf{u}(i)d^*(i) \quad (25)$$

For Recursive Least Square problem, tap weight vector $\mathbf{w}(n)$ can be calculated as

$$\begin{aligned}\Phi(n)\mathbf{w}(n) &= \mathbf{z}(n) \\ \mathbf{w}(n) &= \Phi^{-1}(n)\mathbf{z}(n)\end{aligned}\quad (26)$$

To avoid computationally inefficient calculations of $\Phi^{-1}(n)$ we use a matrix inversion Lemma. The final equation after using the Lemma is

$$\begin{aligned}\Phi^{-1}(n) &= \lambda^{-1}\Phi^{-1}(n-1) - \\ &\frac{\lambda^{-2}\Phi^{-1}(n-1)\mathbf{u}(n)\mathbf{u}^H(n)\Phi^{-1}(n-1)}{1 + \lambda^{-1}\mathbf{u}^H(n)\Phi^{-1}(n-1)\mathbf{u}(n)}\end{aligned}\quad (27)$$

$\mathbf{k}(n)$ is the M-by-1 vector and is called as Gain vector and is defined as the tap input vector $\mathbf{u}(n)$, transformed by the inverse of the correlation matrix $\Phi^{-1}(n)$

$$\mathbf{k}(n) = \Phi^{-1}(n)\mathbf{u}(n)\quad (28)$$

The weight upgradation equation in RLS is obtained

$$\begin{aligned}\mathbf{w}(n) &= \Phi^{-1}(n)\mathbf{z}(n-1) + \Phi^{-1}(n)\mathbf{u}(n)d^*(n) \\ &\quad - \mathbf{k}(n)\mathbf{u}^H(n)\Phi^{-1}(n-1)\mathbf{z}(n-1)\end{aligned}\quad (29)$$

After simplifying, weight vector equation is

$$\mathbf{w}(n) = \mathbf{w}(n-1) + k(n)e^*(n)\quad (30)$$

and error is calculated as

$$e(n) = d(n) - y(n)\quad (31)$$

3. Simulation Results

Table 1. Parameters Used In Simulation

Source Signal Angle	45°
Interference Signal Angle	60°
Signal to Noise Ratio (SNR)	15dB
Signal to Interference Ratio (SIR)	3 dB
LMS Step-size Parameter	0.05
Optimized-LMS Step-size Parameter	0.05
RLS Forgetting Factor	0.999

A linear 8 element array is simulated in MATLAB to compare results of LMS, Optimized-LMS and RLS algorithm. Spacing between adjacent elements of array equals one half of the wavelength.

A simulation run of 100 iterations with parameters summarized in TABLE 1 resulted in error plot as shown in Fig. 2. RLS algorithm showed very fast convergence i-e it took only 3 iterations, Optimized-LMS algorithm converged in 75 iterations where as LMS algorithm exhibited a Brownian motion around minimum value.

To compare the ability of algorithms to give maximum gain in the direction of source signal and placing the null in the direction of interference signal, simulations were performed using different

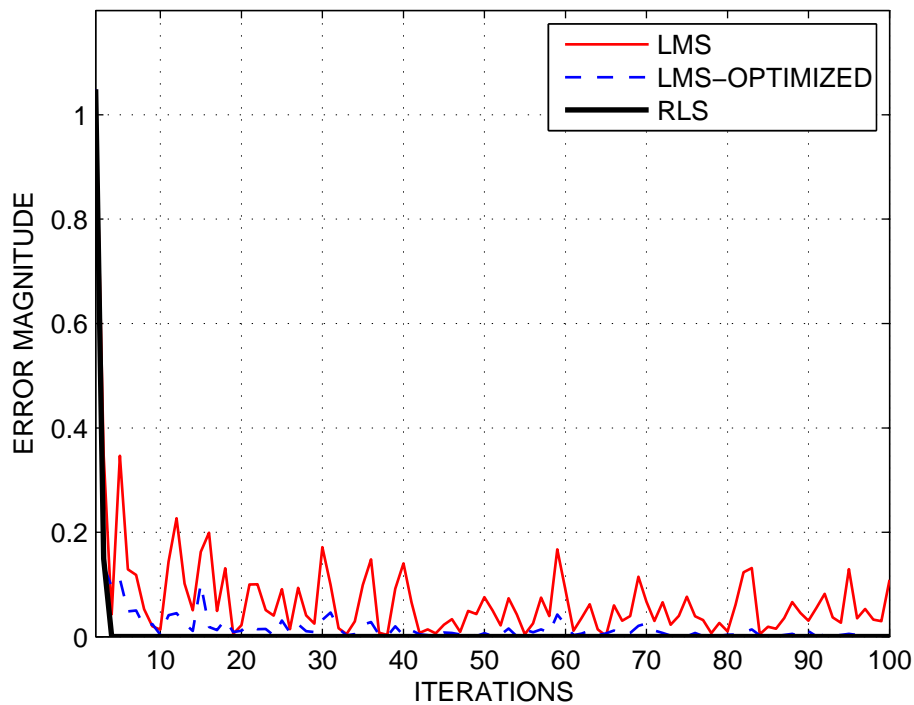


Figure 2. Error plot

interferer signal directions for 500 iterations. Remaining all parameters were set as mentioned above. Fig. 3,4,5 show polar and rectangular plots comparing algorithms for interference signal arrival angle of 40° , 60° and 90° respectively.

For comparing the dependency of algorithms on SNR and SIR, algorithms were compared at two values

- SNR=30dB, SIR=10dB
- SNR=30dB, SIR=10dB

Fig. 6,7,8 show the behavior of LMS, Optimized-LMS and RLS algorithm respectively, for mentioned SNR and SIR values.

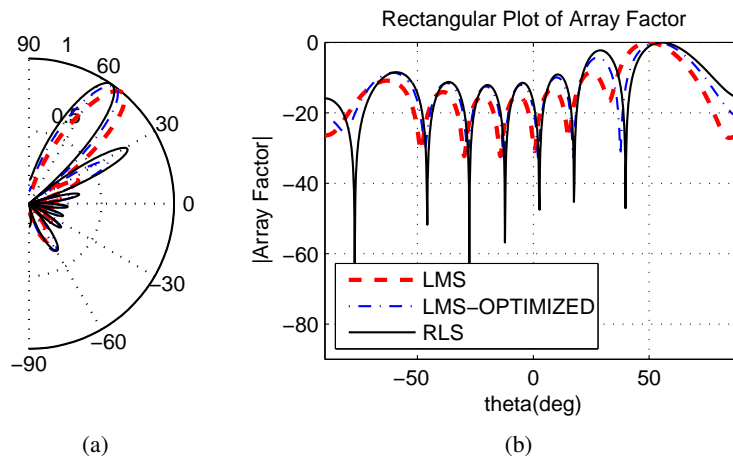


Figure 3. Polar and Rectangular plot with inteference signal at 40°

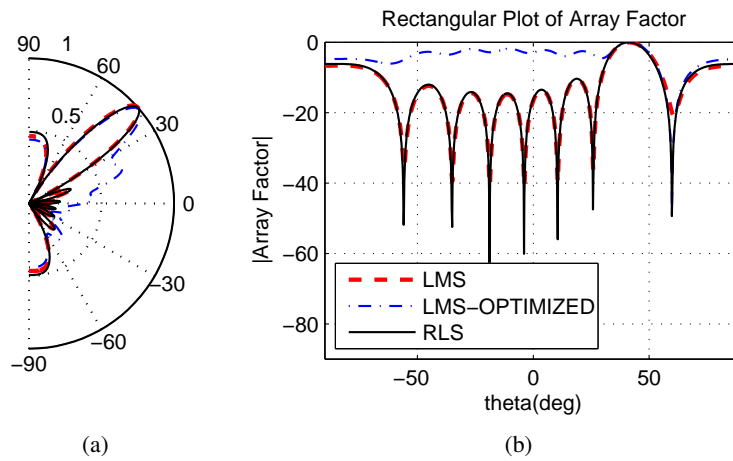


Figure 4. Polar and Rectangular plot with interference signal at 60°

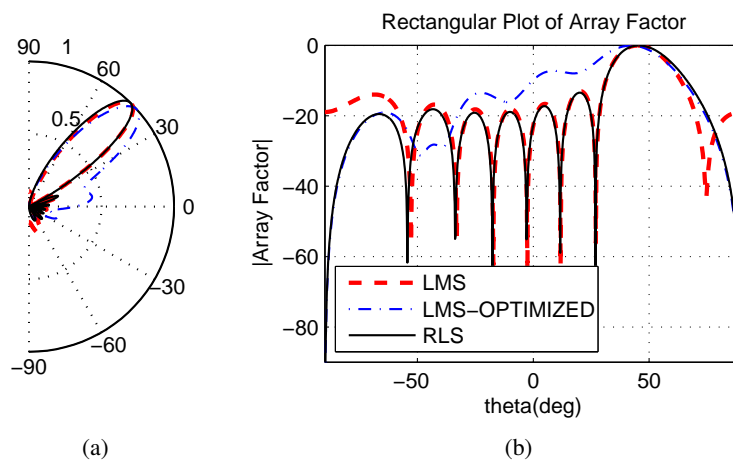


Figure 5. Polar and Rectangular plot with interference signal at 90°

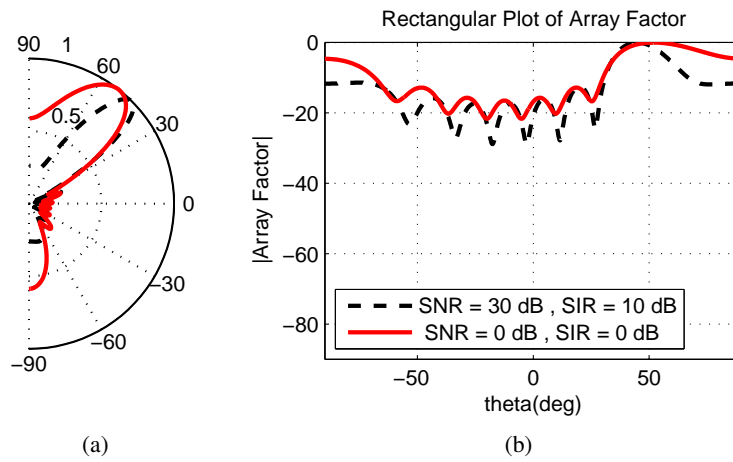


Figure 6. Polar and Rectangular of LMS algorithm

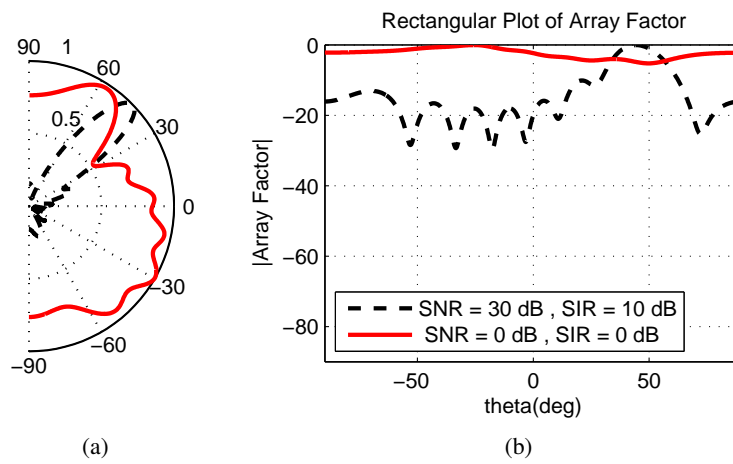


Figure 7. Polar and Rectangular of Optimized-LMS algorithm

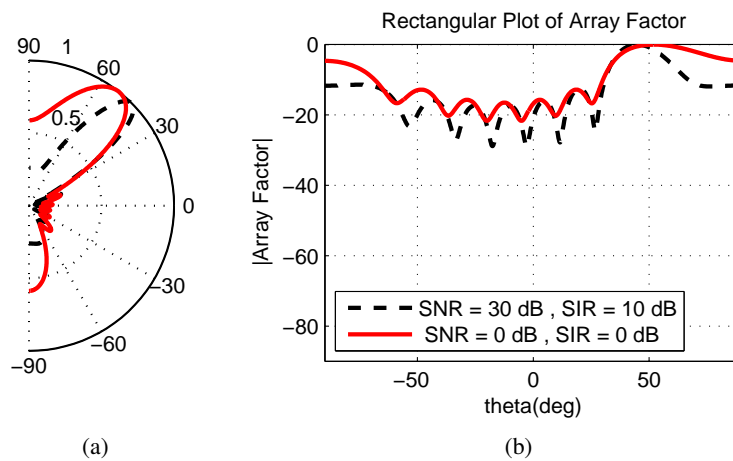


Figure 8. Polar and Rectangular of RLS algorithm

4. Conclusion

By analyzing the graphs, we observed that RLS algorithm provides fastest convergence, Optimized LMS algorithm also shows fast convergence but LMS algorithm lacks the convergence speed. In beamforming results, RLS showed the best beamforming capability placing deeper nulls in case of all the three interference positions i-e 40° , 60° and 90° . The significant difference between the results of LMS and Optimized-LMS in case of beamforming was the presence of many minor lobes in Optimized-LMS. The dependency on SNR and SIR showed that in better conditions i-e high SNR and SIR Optimized-LMS showed the best results. But in poor conditions i-e low SNR and SIR its performance deteriorates. LMS and RLS almost showed equal dependency on SNR and SIR. As the recent developments in digital signal processor (DSP) kits and field-programmable gate arrays (FPGA) have made it possible to implement RLS algorithms in real time systems, and complexity to an extent is not a problem anymore. So RLS algorithm is proposed as it provides deeper nulls in the direction of interferences and faster convergence.

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